EXPANSION OF A GAS CAVITY IN A VARIABLY CONSOLIDATING DILATING MEDIUM

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There are many papers [1-4] on the mechanical effects of a confined explosion in a dilating medium. The distinctive feature here is the effect from the dilatancy, which results in irreversible density change behind the shock-wave front. The expansion found in [2-4] results in a monotone dependence of the residual density on the radius. The density of the medium after the explosion increases monotonically from the walls of the cavity to the periphery. In the calculations of [1-4], the consolidation of the medium at the shock-wave front was assumed constant and independent of the shock-wave intensity, although it is known that the shock consolidation of a porous medium is dependent on the intensity [5]. Calculations have been given [5] on expansion of the cavity in a medium with variable consolidation at the front when there is no dilatancy behind the shock-wave front. In that case, the residual density decreases monotonically away from the walls of the cavity. Here we consider the expansion of a cavity in a medium with variable consolidation at the shock-wave front, and then there is brittle fracture of the medium at the front followed by plastic flow accompanied by dilatancy. We also allow for the escape of gases from the cavity into the pores produced behind the front by the dilatancy. It is found that this reduces the mechanical effect of the underground explosion, i.e., reduces the maximum radius of the cavity and the size of the damage front. It is found that the residual-density profile has a maximum for a variably consolidating dilating medium.

The source of the motion is a spherical cavity of initial radius $a_i$ filled by an expanding gas with adiabatic parameter $\gamma$ and initial pressure $p_0$. At time $t = 0$, a spherical damage wave begins to propagate from the cavity. The initial porosity of the medium is $m_0$. We assume that the Prandtl plasticity condition applies behind the front:

$$\sigma_r - \sigma_\varphi = k + m_1(\sigma_r + 2\sigma_\varphi),$$

where $\sigma_r$ and $\sigma_\varphi$ are the components of the stress tensor and k and $m_1$ are the adhesion coefficient and coefficient of friction, respectively. It is assumed that brittle failure of the medium occurs at the shock-wave front, and therefore the region of plastic flow behind the front will be called the damage zone.

The motion of the medium between the cavity and the wave front is described by the equations for the conservation of momentum and mass together with the dilatancy equation [1]:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} = \frac{1}{\rho}\frac{\partial \sigma_r}{\partial r} + \frac{2}{\rho}(\sigma_r - \sigma_\varphi)/r; \tag{1}$$

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\left(\frac{\partial u}{\partial r} + 2\frac{u}{r}\right) = 0; \tag{2}$$

$$\frac{\partial u}{\partial r} + 2\frac{u}{r} = \Lambda(\rho, \sigma_r)\left|\frac{\partial u}{\partial r} - \frac{u}{r}\right|, \tag{3}$$

where $\rho$ is the density of the medium; u, mass velocity; and $\Lambda$, dilatancy rate. The conditions for conservation of mass and momentum are obeyed at the damage front:

$$u(R) = \varepsilon(R)\dot{R},\ \sigma_r(R) = -\rho_0\varepsilon(R)\dot{R}^2 - \sigma^*,$$

where $\sigma^*$ is the stress at which irreversible damage occurs, $\dot{R}$ is the speed of the shock-wave front, and $\varepsilon(R)$ is the consolidation at the front, which is

$$\varepsilon(R) = 1 - \rho_0/\rho(R) = \varepsilon_0(a_i/R)^\lambda,\ \lambda > 0. \tag{4}$$

Expression (4) was used in [5]. In terms of Lagrange variables, (1)-(3) become

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TABLE 1

| Variant No. | Λ | λ | Expansion of gases | $\varepsilon_0$ | Variant No. | Λ | λ | Expansion of gases | $\varepsilon_0$ |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0,2 | 0 | *H* | 0,1 | 6 | 0,2 | 0,5 | *H* | 0,1 |
| 2 | 0,2 | 0,5 | *H* | 0,1 | 7 | 0,07 | 1 | *H* | 0,1 |
| 3 | 0,2 | 0 | *A* | 0,1 | 8 | 0 | 0 | *A* | 0,2 |
| 4 | 0,07 | 1 | *H* | 0,1 | 9 | 0,2 | 0 | *A* | 0,2 |
| 5 | 0 | 0,5 | *H* | 0,1 | | | | | |

$$\rho_0 r_0^2 r^{\alpha-2}\frac{\partial u}{\partial t}=\frac{\partial}{\partial r_0}\left[r^{\alpha}\left(\sigma_r(r)+\frac{k}{3m_1}\right)\right]; \tag{5}$$

$$\frac{\partial r}{\partial r_0}=\frac{r_0^2}{r^2}\frac{\rho_0}{\rho}; \tag{6}$$

$$\Lambda\frac{\partial}{\partial t}\ln(\rho r^3)+\frac{\partial}{\partial t}\ln\rho=0, \tag{7}$$

where $\alpha = 6m_1/(2m_1 + 1)$; $p(r_0, t) = -\sigma_r(r_0, t)$, and in (5)-(7) we transfer to dimensionless variables by using the following formulas: $x = a/a_i$, $R' = R/a_i$, $\tau = t\sqrt{p_0/\rho_0}/a_i$, $p' = p/p_0$, $\rho' = \rho/\rho_0$, $\sigma_r' = \sigma_r/p_0$, $\sigma_\varphi'=\sigma_\varphi/p_0$, $r' = r/a_i$, $r_0' = r_0/a_i$. Behind the shock-wave front in a medium whose plastic flow involves dilatancy, the density is less than that at the front [2]. In other words, the dilatancy results in an increase in the porosity after the passage of the wave. Experiment [6] shows that the gas can penetrate from the cavity into the holes formed behind the front even during the dynamic stage of the explosion. Penetration of the gas from the cavity into the pores may [6] reduce the performance of the explosion by reducing the gas pressure in the cavity. Also, gas adsorption at the leading edges of the cracks may reduce the effective Griffiths porosity. The effective strength of the medium may also be reduced by the finite gas pressure in the pores. An independent study is required to examine the simultaneous effects of all these factors. Here we consider only the escape of gas from the cavity on the assumption of instantaneous pore filling.

We assume that the gas in the cavity expands in accordance with the law $pV_{\text{eff}}^{\gamma} = \text{const}$, where $V_{\text{eff}}$ is the volume of the cavity plus the volume of the pores formed in the medium after passage of the damage wave as a result of the dilatancy (it is assumed that the gases can enter these pores from the cavity without hindrance). Therefore, $p_0\left(\frac{4\pi}{3}a_i^3\right)^{\gamma}=-\sigma_r(a)\left(\frac{4\pi}{3}a^3+V_f\right)^{\gamma}$, where $V_f$ is the volume of the pores filled by gas from the cavity. In dimensionless form, $\sigma_r'(x)=-(x^3+v_f)^{-\gamma}$, $v_f=V_f\Big/\frac{4\pi}{3}a_i^3$. We integrate (2) with allowance for the spherical symmetry with $\Lambda = \text{const}$ to get $u(r) = c(t)/r^n$, and from the boundary conditions we get $u(r) = a^n\dot{a}/r^n$ [2], $n = (2 - \Lambda)/(1 + \Lambda)$; the value of $V_f(t)$ was calculated as follows. The volume of the pores in a spherical layer of radius r and thickness dr is $dV_f = 4\pi r^2 dr m(r)$, where $m(r)$ is the porosity at distance r from the center of the explosion, which is $m(r) = 1 - (1 - m_0)\rho/\rho_0$; integration and conversion to dimensionless form give

$$v_f=R'^3-x^3-3(1-m_0)\int_x^{R'}\rho'(r')r'^2dr'.$$

The expression for $\rho'(r'(r_0', \tau))$ may be found by analogy with [3]. Calculation gives

$$\rho'(r'(r_0',\tau))=\frac{1}{1-\varepsilon_0 r_0'^{-\gamma}}\left(\frac{r_0'}{r'}\right)^{2-n}.$$

To calculate the time course of the cavity and damage wave we integrate (5) with $r_0'$ and $R'(\tau)$ as limits; we use the boundary conditions and the fact that the gas in the cavity does not behave adiabatically. We finally get for the confined explosion that

$$dy/dx+N(x)y=M(x), \tag{8}$$

where $y = \dot{x}^2$; $\ddot{x} = (1/2)dy/dx$; $y(x = 1) = \varepsilon_0$,

$$M(x)=2\,\frac{(R'^{\alpha}-x^{\alpha})\frac{1}{3m_1}\frac{k}{p_0}-x^{\alpha}\sigma_r'(x)-R'^{\alpha}\sigma^*/p_0}{x^nY};$$

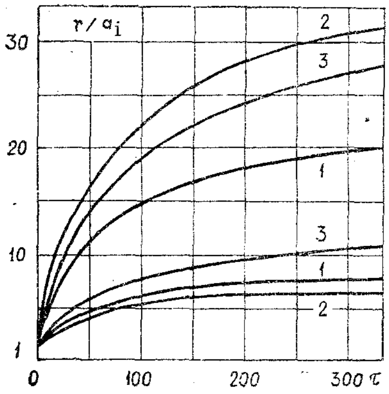

Fig. 1.

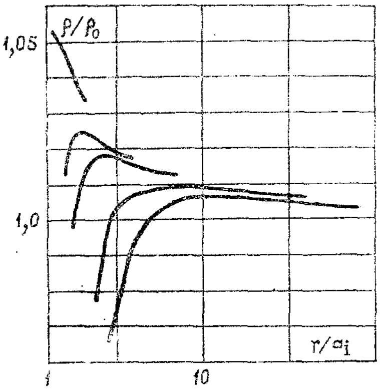

Fig. 2.

$$N(x)=\frac{2n}{x}-2\frac{x^{n}\left[nX-R'^{\alpha-2(n-\lambda)}\varepsilon_0^{-2}\varepsilon(R')\right]}{Y};$$

$$X=\int_{1}^{R'} r_0'^{2} r'^{\alpha-3-2n}(r_0')\,dr_0';\quad Y=\int_{1}^{R'} r_0'^{2} r'^{\alpha-2-n}(r_0')\,dr_0'.$$

The general solution to (8) is found by varying the constants:

$$y(x)=\left\{\varepsilon_0+\int_{1}^{x}\left[M(x')\,e^{\int_{1}^{x'}N(x'')dx''}\right]dx'\right\}e^{-\int_{1}^{x}N(x')dx'}. \tag{9}$$

The final solution to (9) was obtained numerically by computer.

We now discuss the results of the numerical solution. Table 1 gives the sets of parameters used in the calculations. Here A corresponds to adiabatic expansion of the explosion gases, while in the case denoted by H the gases may penetrate into the pores, with $\varepsilon_0$ the consolidation at the front (for $\lambda = 0$) or else the consolidation at the front at the initial instant (for $\lambda > 0$). The following values for the constants were used in all cases: $p_0 = 0.7\cdot 10^8$ kPa, $a_i = 3$ m, $\gamma = 1.4$, $m_1 = 0.1$, $|k| = 10^3$ kPa, $\sigma^* = 10^4$ kPa, $\rho_0 = 3.5$ g/cm$^3$, $m_0 = 0.2$.

Figure 1 shows $x(\tau)$ and $R'(\tau)$ for a medium with dilatancy. Here and subsequently the numbers on the curves correspond to the numbers of the sets of parameters in Table 1. Note that curves 1 and 2 for $x(\tau)$ and $R'(\tau)$ have been constructed for the cases where allowance is made for escape of the gases from the cavity during expansion. In the case of curve 1, the compression at the front is taken as constant ($\lambda = 0$), while $\lambda = 1/2$ for curve 2, i.e., the consolidation at the front is variable and is $\varepsilon(R') = \varepsilon_0(1/R')^{1/2}$; comparison of curves 1 and 2 for $x(\tau)$ and $R'(\tau)$ shows that an increase in $\lambda$ results in an appreciable reduction in the size of the cavity and a marked increase in the radius of the damage zone, which occurs because there is a reduction in the proportion of the energy dissipated by collapse of the pores in the case of decreasing consolidation at the front. The size of the cavity is reduced because in our model the cavity expands at the expense of the collapsing pores. When the consolidation decreases away from the center, a smaller proportion of the pore volume collapses, which results in a reduction in the cavity size. Curves 1 and 3 for $x(\tau)$ and $R'(\tau)$ correspond to a dilating uniformly consolidating medium. For the case corresponding to curve 1, gases penetrate from the cavity into the pores formed at the stage of expansion, while curve 3 corresponds to the gases in the cavity expanding adiabatically. The escape of gas from the cavity reduces $x(\tau)$ and $R'(\tau)$, i.e., reduces the mechanical effects of the explosion. These results have been confirmed by experiment [6].

The curves in Fig. 2 show the time course of the spatial profile in the density for various instants. The curves in Fig. 2 correspond to model 4. Figure 2 shows that at $\tau_1 = 0.4$ the density $\rho'(r')$ decreases with the distance from the wall of the cavity for all $r'$. The curve corresponding to time $\tau_2$ ($\tau_2 = 2.2$) has a maximum, while on the third curve ($\tau_3 = 4.6$) the density at the cavity wall is below the background on account of the dilatancy. As

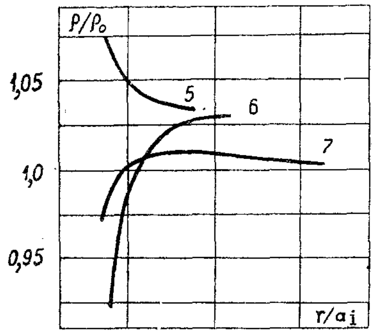


Fig. 3.

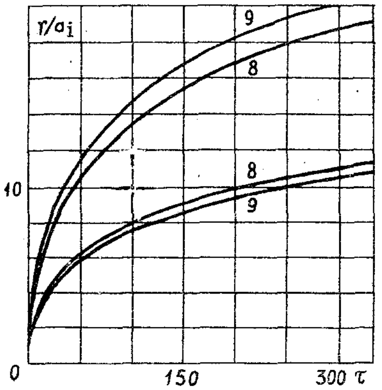


Fig. 4.

time passes, the maximum does not disappear but becomes less steep and retreats from the cavity wall.

Figure 3 shows the mechanism responsible for this maximum. All curves have been referred to the time $\tau = 18.1$, while the gases in the cavity are nonadiabatic. In the absence of dilatancy, the consolidation at the shock front decreases away from the center, and the density therefore decreases (curve 5). In the case of strong dilatancy ($\Lambda = 0.2$) and slightly variable consolidation at the front (curve 6), the density decreases, the density at the cavity wall being less than the background density, but with an increase away from the center. The competing dilatancy and consolidation decreasing with distance from the center result in a nonmonotone radial dependence of the density, with a maximum (curve 7). The qualitative picture given by Fig. 3 persists as time passes.

Figure 4 shows the effects of the dilatancy on the dimensions of the cavity and damage zone. Both curves correspond to adiabatic expansion of the gas in the cavity and constant consolidation at the shock front. Curve 8 corresponds to flow without dilatancy behind the front ($\Lambda = 0$), while 9 corresponds to flow with dilatancy ($\Lambda = 0.2$). The dilatancy clearly increases the size of the damage zone but reduces the radius of the explosion cavity. This result corresponds to that in [3].

In conclusion, we note that a nonmonotone course of $\rho(r)$ can be obtained when one incorporates the dependence of the dilatancy rate $\Lambda(m_0, \sigma_r(R))$ on the porosity and the pressure, as V. K. Sirotkin has pointed out to us.

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